

01.07.2024 Cash Award Rider - Author's Solution

Given :

$$AM^2 + AN^2 = BM^2 + BN^2 = CM^2 + CN^2 = DM^2 + DN^2$$

Construction :

Join MN, Let O be midpoint of MN. Join OA, OB, OC & OD.

In $\triangle BMN, \triangle AMN, \triangle DMN$ & $\triangle CMN$, BO, AO, DO & CO are medians.

By Apollonius Theorem,

$$AM^2 + AN^2 = 2(OA^2 + OM^2) \text{ -----(1)}$$

$$BM^2 + BN^2 = 2(OB^2 + OM^2) \text{ -----(2)}$$

$$CM^2 + CN^2 = 2(OC^2 + OM^2) \text{ -----(3)}$$

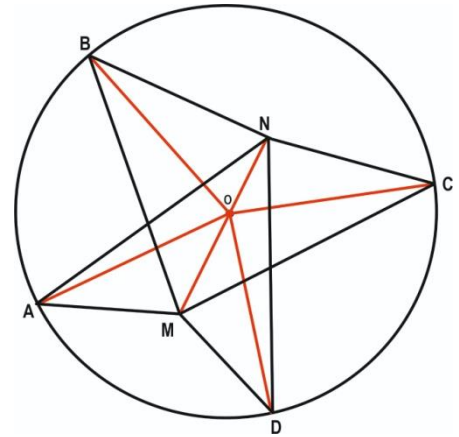
$$DM^2 + DN^2 = 2(OD^2 + OM^2) \text{ -----(4)}$$

(1), (2), (3) & (4) \rightarrow

$$\Rightarrow OA^2 = OB^2 = OC^2 = OD^2 \Rightarrow OA = OB = OC = OD$$

A circle drawn with centre O and radius OA will pass through A, B, C & D.

Hence, ABCD is concyclic



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